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Mathematical Model for Electroosmotic Dewatering of Activated Sludge*

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Abstract：The mechanical dewatering of activated sludge is difficult due to its high compressibility, which can be improved by electroosmosis. In electroosmosis, direct electric field is applied to sludge cake. Based on the conductivity modes of different sludge beds, a model is presented in which sludge cake consists of two series parts in the circuit: a dewatered bed and an undewatered one. The dewatered bed called solid conductor is mainly made up of immovable water and sludge particles. The undewatered bed includes movable water and solid conductor, which are connected in parallel in the circuit. The model describes the variation of water content with time and electric power consumption as a function of water content in sludge cake, and interprets the reason for the variation of electroosmotic dewatering rate. Comparison with the experimental data for electroosmotic dewatering under constant voltage supports the validity of the model.
Keywords：electroosmotic dewatering; activated sludge; conductivity; electric power consumption

Sludge production from wastewater treatment plants has been increasing in recent years in China, bringing lots of problems to city environment. Although sludge had been flocculated and mechanically dewatered, the water content in sludge cake was 80% (mass ratio) or higher, whereas water content in dewatered sludge cake should be less than 60% for disposal[1]. Thus, further dewatering is necessary. Currently, the thermal drying method is often considered sufficient to remove capillary and vicinal water. However, capital and operating costs of this method are high[2]. Electroosmosis technique offers a potential cost-effective solution, which can be applied to remove the capillary and vicinal water. In addition, the process does not involve phase change, thus making it more favorable because of its lower energy consumption than thermal drying[3].

Electroosmotic dewatering is based on the surface charge of sludge and applied electric field. The sludge particles with a negative surface are surrounded by a layer of positive charges, forming electric double layers. When an electric field is applied to sludge cake, the particles stay trapped in the cake matrix, but the positive counterions are attracted by the cathode. As they move towards the cathode, they drag and push the water molecules in the pores, resulting in a net transport of water towards the cathode[4,5].

Electroosmotic dewatering has been widely studied, but the modelling of the process has received little attention. Curvers *et al*[4] proposed a model which described the height of compressible materials as a function of time in a period when pressure dewatering was assisted by a DC electric field. Iwata *et al*[6] developed a model for electroosmotic dewatering of a filter cake that is assumed to possess a homogeneous porosity distribution after 24 h preconsolidation, but the model cannot be adopted for realistic dewatering, involving the simultaneous action of filtration and expression as well as electroosmotic dewatering in a single process. Yoshida *et al*[7] constructed the design equations for electroosmotic dewatering and analyzed theoretically the mechanism based on electroosmotic flow through a compressible packed bed of particles.

This paper presents a model based on the conductivity mode of sludge cake by electroosmotic dewatering. The electric field is applied to dewatered sludge cake by mechanical means, as it is economically favorable[8]. Modelling the overall process is important for understanding the basis of electroosmosis and optimizing the process parameters such as the time at which the electric

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field is applied.

1 Mathematical model

Sludge cake being dewatered by electroosmosis can be approximately represented by a simple model, as shown in Fig.1. Some assumptions are made as follows:

(1) The compressibility of dewatered sludge cake by mechanical method is neglected.

(2) All sludge particles stay trapped in the cake matrix and they will not run off.

(3) Electrophoresis—the movement of particles within liquid under an electric field—can be assumed to be nonexistent, and hence only electroosmosis needs to be taken into account.

(4) The electrolysis reactions at the electrodes and temperature have no impact on the conductivity and viscosity of water in the pores.

(5) Under electric field strength or current intensity, there is a corresponding state for a given sludge bed by electroosmosis, that is, the total water is assumed to be made up of immovable water and movable water, and electroosmotic dewatering is only effective for movable water.

(6) During electroosmotic dewatering, sludge cake in this model is divided into two parts: in one part dewatering has finished, and in the other dewatering is proceeding. As shown in Fig.1, the upper part Ⅰ is a dewatered sludge bed with a terminal water content corresponding to the applied electric field, from which all liquid flows out, and immovable water is left only in this part. The lower part Ⅱ is an undewatered sludge bed with the same water content as initial water content, and the liquid-filled pores are idealized as cylindrical capillaries.

(7) From the viewpoint of conductivity mode, the conductor of sludge cake is composed of conductor Ⅰ and conductor Ⅱ, which are series in the circuit. Conductor Ⅰ is called solid conductor because it is mainly made up of immovable water and sludge particles, and immovable water only plays an electric conduction role. Conductor Ⅱ includes the conductor of movable water by electroosmosis which is connected with solid conductor in parallel in the circuit. A sketch of conductivity mode of sludge cake during electroosmotic dewatering is shown in Fig.2, where R1 denotes conductor Ⅰ of Fig.1, R2 the solid conductor (immovable water and sludge particles) of conductor Ⅱ, and R3 the conductor of movable water of conductor Ⅱ.

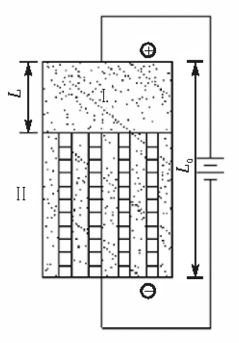

Fig.1 Schematic diagram of electroosmotic dewatering model of sludge cake

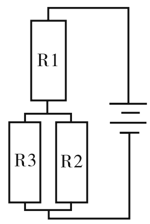

Fig.2 Conductivity mode of sludge cake

During electroosmotic dewatering of sludge cake, all solid particles stay trapped in the cake matrix. Taking the material balance of water in sludge cake into account, the increase in the volume of part Ⅰ equals the volume of water removed, Q, hence

$$Q = LA \tag{1}$$

where A is the cross-sectional area of sludge cake; and L the thickness of the dewatered part at time t. It is clear that the value of L changes with dewatering time. The average water content x in the whole sludge cake is given by

$$x = x_0 - \frac{L}{L_0}(x_0 - x_c) \tag{2}$$

where x_0 is the initial water content of sludge cake; x_c the terminal water content of sludge cake; and L_0 the initial thickness of sludge cake. In electroosmotic dewatering, it is impossible in principle to achieve complete water removal in sludge cake, when a liquid state in sludge cake becomes not continuous gradually with the proceeding of dewatering, sludge does not conduct electricity and then electroosmosis is no longer caused[9], so the value of x_c for given sludge is constant, which depends on the sludge properties and the applied electric field.

R1 and R2 are both assumed to be solid conductor, because they are composed of conductivity of immovable water and particles in sludge cake, and they behave as solid. During electroosmotic dewatering, if the operating conditions are defined, the properties of R1 and R2 are the same, so the sum of electric resistance values of R1 and R2 is a constant.

$$R_0 = R_1 + R_2 \tag{3}$$

where R_1 and R_2 are the electrical resistance of R1 and R2, respectively. According to Eq.(2), the expressions for the electrical resistance of R1 and R2 can be written as

$$R_1 = R_0 \frac{L_a}{L_0} = R_0 \frac{x_0 - x}{x_0 - x_c} \tag{4}$$

$$R_2 = R_0 \frac{L_b}{L_0} = R_0 \frac{x - x_c}{x_0 - x_c} \tag{5}$$

where L_a is the thickness of dewatered part Ⅰ in Fig.1; and L_b the thickness of dewatering part Ⅱ.

Supposing the initial value of electrical resistance of movable water in sludge cake before elctroosmotic dewatering is R_0', the electrical resistance of R3 can be expressed as

$$R_3 = R_0' \frac{L_b}{L_0} = R_0' \frac{x - x_c}{x_0 - x_c} \tag{6}$$

According to the conductivity modes of different sludge beds, the total electric resistance R of the circuit is given by

$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3} \tag{7}$$

Substituting Eqs.(4), (5) and (6) into Eq.(7),

$$R = R_0 \frac{x_0 - x}{x_0 - x_c} + \frac{R_0 R_0'}{R_0 + R_0'} \cdot \frac{x - x_c}{x_0 - x_c} \tag{8}$$

When a constant voltage V_0 is applied to sludge cake, based on Ohm's law, the current through sludge cake can be written as

$$I = \frac{V_0}{R} = \frac{(R_0 + R_0')(x_0 - x_c)V_0}{R_0^2 x_0 + R_0 R_0'(x_0 - x_c) - R_0^2 x} \tag{9}$$

Hence the effective electric field strength for undewatered part Ⅱ is given by

$$E_e = \frac{I \frac{R_2 R_3}{R_2 + R_3}}{L_b} = \frac{\frac{R_0 R_0'}{(R_0 + R_0')L_0}(x_0 - x_c)V_0}{\left(R_0 x_0 - \frac{R_0 R_0'}{R_0 + R_0'} x_c\right) - \frac{R_0^2}{R_0 + R_0'} x} \tag{10}$$

From Eq.(9) and Eq.(10), it can be seen that the current and effective electric field strength for electroosmotic dewatering both decrease with the decrease of x. When a voltage is applied to sludge cake, water in sludge cake is removed downward. Electroosmotic dewatering rate v can be calculated by Helmholtz-Smoluchowski equation[10]:

$$v = \frac{\varepsilon \varsigma}{4\pi\eta} E \tag{11}$$

where ε is permittivity of the solution; ς zeta potential on the particle surface; and η viscosity of the solution. Electroosmotic dewatering rate v is proportional to the electric field strength of E. Let

$$\kappa = \frac{\varepsilon \varsigma}{4\pi\eta} \tag{12}$$

where κ is called electroosmotic dewatering coefficient, which may be obtained directly from experiment or calculated according to Eq.(12).

Combining Eq.(1) and Eq.(2), the average water content during electroosmotic dewatering can be written as

$$x = x_0 - \frac{(x_0 - x_c)}{L_0} \int_0^t v \, dt \tag{13}$$

Substituting Eq.(10) and Eq.(11) into Eq.(13), an integral equation is derived as

$$x = x_0 - \frac{(x_0 - x_c)}{L_0} \cdot \int_0^t \frac{\kappa \frac{R_0 R_0'}{(R_0 + R_0')L_0}(x_0 - x_c)V_0}{\left(R_0 x_0 - \frac{R_0 R_0'}{R_0 + R'} x_c\right) + \left(\frac{R_0 R_0'}{R_0 + R'} - R_0\right) x} dt \tag{14}$$

Differentiating both sides of Eq.(14) with respect to parameter t,

$$\frac{dx}{dt} = -\frac{(x_0 - x_c)}{L_0} \cdot \frac{\kappa \frac{R_0 R_0'}{(R_0 + R_0')L_0}(x_0 - x_c)V_0}{\left(R_0 x_0 - \frac{R_0 R'}{R_0 + R_0'} x_c\right) + \left(\frac{R_0 R'}{R_0 + R_0'} - R_0\right) x} \tag{15}$$

Let

$$K_1 = R_0 x_0 - \frac{R_0 R_0'}{R_0 + R_0'} x_c$$

$$K_2 = \frac{R_0 R_0'}{R_0 + R_0'} - R_0$$

$$K_3 = \frac{R_0 R_0'}{R_0 + R_0'}$$

where K_1, K_2 and K_3 are constant, and can be obtained directly from experiment. So a mathematic model, describing the relationships among the average water content in sludge cake, the initial electric field strength, the thickness of sludge cake and dewatering time, can be obtained as

$$x=\frac{-K_1+\sqrt{K_1^2-2K_2\left[K_3\kappa(x_0-x_c)^2x_0\frac{V_0}{L_0}\frac{t}{L_0}-(K_1x_0+\frac{1}{2}K_2x_0^2)\right]}}{K_2} \quad (16)$$

If other operating parameters are kept constant, the average water content is proportional to the square root of time.

In practice, water content in sludge cake is an important factor, so the reliability of Eq. (16) should be proved via experiment.

Electroosmotic dewatering technique offers a promising approach to sludge dewatering. In practice, however, electroosmotic dewatering has to compete with post-dewatering drying techniques, and energy efficiency plays a major role in the successful full-scale implementation. Therefore, investigating the electric energy requirements is important. The energy consumption can be calculated by

$$P=\frac{1}{m_t}VIt \quad (17)$$

where P is energy consumption per unit mass of water removal, kW·h/kg; m_t mass of water removal, kg; V the applied voltage, V; t the processing time, h; and I the current at time t through the whole sludge cake, A. So substituting Eqs. (1), (2) and (9) into Eq. (17),

$$P=\frac{(R_0+R_0')(x_0-x_c)^2V_0^2t}{L_0A(x_0-x)(R_0^2x_0+R_0R_0'(x_0-x_c)-R_0^2x)}+C \quad (18)$$

where C is a corrected parameter, reflecting the influence of thermal effect of sludge cake and electrolysis at electrodes in practice on the energy consumption during electroosmotic dewatering, and it can be obtained from experiment.

In economical evaluation and design for the equipment, the electric energy consumption per unit mass of water removal P is significant, so Eq. (18) is also one of the main model and it will be verified via the experiment.

2 Validation of the model

To validate the model, experiments were performed under rent electric fields. The results from one of these experiments were used to determine the values of the model parameters. These values were then used in the model to predict the water content in sludge cake as a function of time (Eq. (16)) and energy consumption as a function of water content (Eq. (18)) during electroosmotic dewatering. The prediction results are compared with the experimental results.

2.1 Materials and methods

The experimental set-up is shown in Fig.3. The acrylic cylindrical cell (internal diameter of 0.07 m) was positioned vertically. A couple of stainless steel electrodes were arranged horizontally so that electric field was in vertical direction. The anode was set to contact the top surface of sludge cake, and a pressure of 7 kPa was applied at the same time to ensure a close electric contact between the anode and sludge. The cathode was placed downstream of sludge cake. A piece of sponge placed underneath the cathode and contacting the cathode closely, was utilized to absorb water flowing out from the cathode.

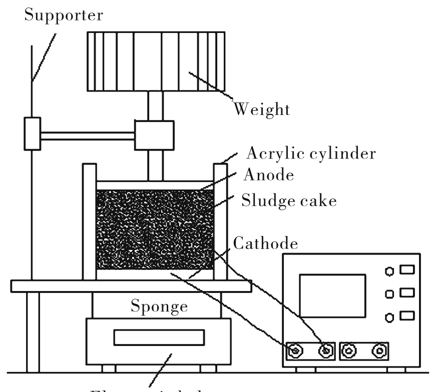

Fig.3 Vertical experimental set-up of electroosmotic dewatering

Sludge cake adopted in this research was obtained from Tianjin Jizhuangzi Wastewater Treatment Plant in China. This plant treats municipal sewage and has a capacity of 260 000 m^3 of sewage production per day. Sludge from this sewage treatment plant was activated and thickened. During the thickening process, a certain amount of cationic polyelectrolyte was added to the sludge for coagulation and flocculation, and sludge was dewatered by centrifugation. Dewatered sludge cake was then collected and characterized. Sludge cake had a pH value of 7.58, an initial water content of 79.0% (mass ratio) and an organic content of 63.9% (mass ratio).

Sludge cake was packed into the cylindrical cell, and the initial height of sludge cake was 5 mm. A DC electric field was applied and the experiments were carried out under constant voltage. The mass of water absorbed by the sponge was measured at different time. In the meantime, the electric current passing through sludge cake was recorded at intervals during the elctroosmotic dewatering. Each experiment under a certain operation was triply repeated.

Water content w in sludge cake is calculated by

$$w = \frac{m_0 \cdot w_0 - m_t}{m_0 - m_t} \times 100\% \quad (19)$$

where m_0 is the overall mass of sludge at the initial stage; w_0 the initial water content of sludge cake; and m_t the mass of water collected at time t.

2.2 Results and discussion

According to Helmholtz-Smoluchowski equation (Eq. (11)), it is perceived that electroosmotic dewatering rate v for a given porosity under constant electric field strength keeps steady, but it declines in practice. This may be that electric field strength is not uniform across sludge cake, as well as property changes caused by electric contacts at the electrode such as pH, porosity and zeta potential[11]. In our previous assumption, the properties of sludge particles and water solution do not change, so the decline of electroosmotic dewatering rate is mainly due to the non-uniform distribution of electric field strength. We suppose that electroosmotic dewatering only occurs in part Ⅱ when the water content of sludge cake in part Ⅰ reaches the terminus, and sludge cake in part Ⅰ plays a role in conducting electricity. So sludge cake in part Ⅰ consumed a part of the applied voltage, resulting in the decrease of effective electric field strength in part Ⅱ.

Effective electric field, as a driving force of electroosmotic dewatering, was calculated by Eq. (10) using the parameters such as R_0, R_0', x_0, x_c and L_0. According to experimental data, the parameters R_0, R_0', x_0 and x_c could be easily obtained, which were 328.95 Ω, 9.36 Ω, 79% and 56%, respectively. Effective electric field strength across un-dewatered sludge cake as a function of the average water content in sludge cake is shown in Fig.4.

Fig.4 demonstrates that effective electric field strength decreases drastically with the decrease of water content in sludge cake. Because of the drastic drop of effective electric field strength across un-dewatered sludge cake, electroosmotic dewatering becomes more and more difficult. If the expression for the effective electric field strength is correct, the relationship between electroosmotic dewatering rate and effective electric field strength should be a linear function according to Helmholz-Smoluchowski equation. To verify the relation, dewatering rate measured experimentally as a function of effective electric field strength is shown in Fig.5, which indicates that dewatering rate increases linearly with effective electric field strength except the first three data which were not employed in Fig.5. Therefore, the assumptions are appropriate based on the conductivity modes of different sludge beds during electroosmotic dewatering.

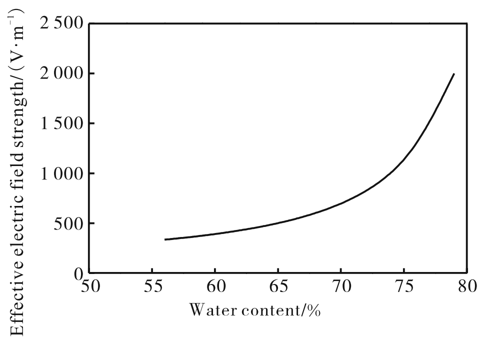

Fig.4 Change of effective electric field strength with average water content

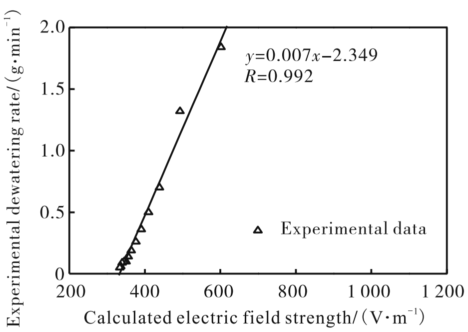

Fig.5 Relationship between dewatering rate and effective electric field strength

Under constant voltage, the change of the average water content in sludge cake with time is illustrated in Fig.6. According to the experimental data, the parameters K_1, K_2 and K_3 could be easily obtained, which were 25 81.4, −319.9 and 9.1, respectively. Using K_1, K_2 and K_3, as well as the presented mathematical model (Eq. (16)), the average water content in sludge cake was calculated by

$$x = 79.654 - 6.719\sqrt{0.06 + t} \quad (20)$$

Theoretical and experimental values under different time are also shown in Fig.6. The results indicate that the theoretical data are basically in accordance with the experimental data. In the meantime, water content drops significantly at the beginning; however, with the increase of dewatering time, the slope of water content-time curve becomes smaller and smaller.

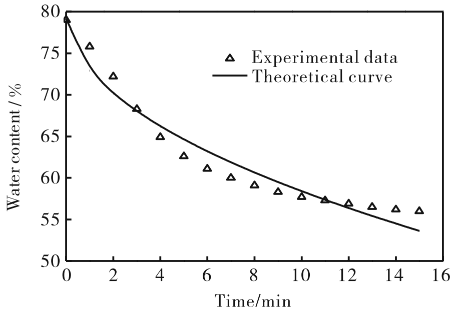

Fig.6 Comparison between theoretical and experimental water content as a function of time

Fig.7 shows the comparison between theoretical and experimental energy consumption as a function of water content in sludge cake during electroosmotic dewatering. The corrected parameter C in Eq.(18) is 0.035 6. It can be seen that the theoretical values are consistent with experimental data, and the energy consumption increases sharply when water content in sludge cake decreases to some extent.

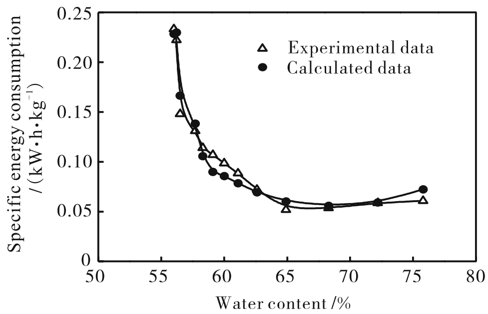

Fig.7 Comparison between theoretical and experimental energy consumption as a function of water content in sludge cake

3 Conclusions

The electroosmotic dewatering model under constant voltage is developed from conductivity mode of sludge cake. The model describes the variation of water content in sludge cake with time, and the energy consumption as a function of water content during electroosmotic dewatering. By the proper determination of the model parameters, experimental data are in good agreement with theoretical values. This model is applicable to the prediction and optimization of practical operation, particularly for mechanically dewatered sludge cake. Meanwhile, it can explain why the dewatering rate becomes slower and slower with time.

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